Sizing Considerations for Electrodynamic-Tether-Driven Atmospheric Harvestor

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Introduction

This notebook presents a process for determining some of the factors that would go into the design of an atmospheric harvester. The focus is in the electrodynamic drive and power array. The study presents very little on the mechanics of the harvester itself. The altitude of the collector is determined by thermal considerations of the cable rather than the gas dynamics of collection. The goal is to give an idea of the scale of the total mass needed to achieve a mass collection goal. We have arbitrarily set a goal of 1000 kg/day of collected gas (total of oxygen plus nitrogen).

The basic idea of the haves tor is to drag a collection device through the upper atmosphere. An electrodynamic cable is used to balance the drag forces. A current flow through the cable interacts with the earth’s magnetic field to provide a propulsion force. The system is powered by a solar array at the upper end of the cable.

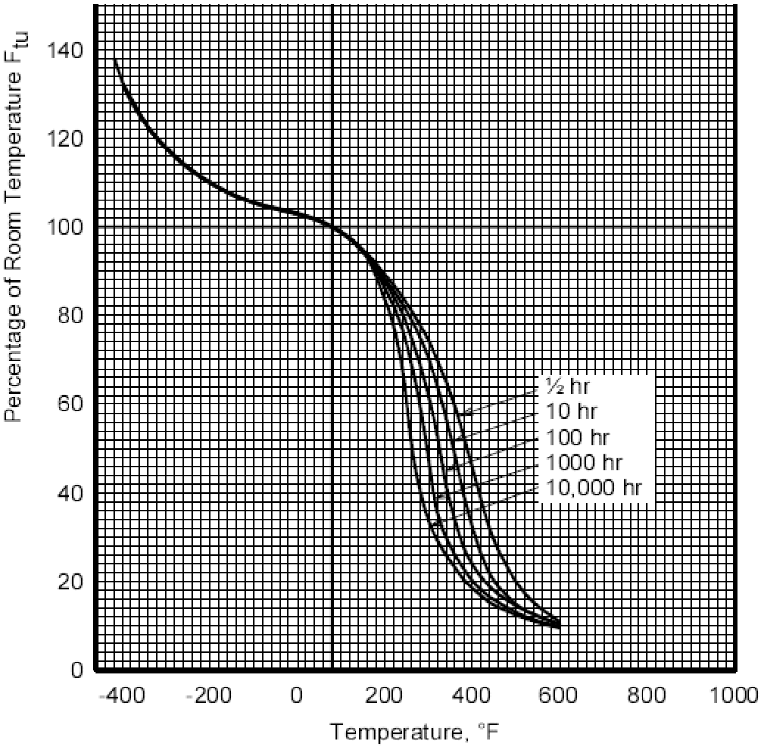
Miscellaneous Data

Cable Material Properties

The drive cable will be made either entirely from aluminum alloy, or an aluminum steel mix. The material selection is based mostly on the good electrical conductivity of aluminum relative to it’s density. The disadvantage of aluminum is it’s relatively low operational temperature. One goal of the study is to demonstrate that a system can be designed even with modest temperature limits. In order to operate at lower altitudes, steel can be added to the cable (as separate strands, not an alloy). The composite mixture can have good conductivity as well as adequate strength as higher temperature.

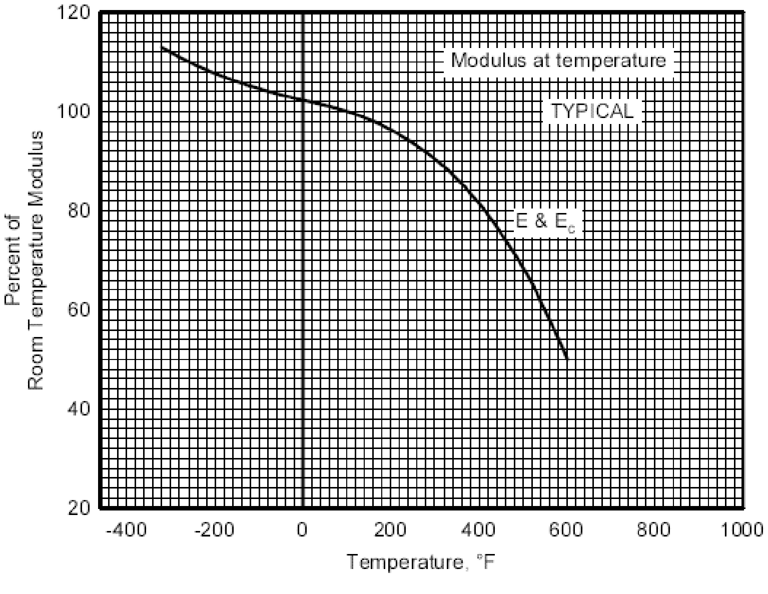
For some quick numbers we use material properties for aircraft structures without regard for the material forms. In other words the properties may be for a plate form while the cable would be a drawn strand.

For aluminum, assume 7075 alloy and use the following graph for strength retention. Assume 10,000 hr exposure is close to infinite time.



Ultimate Tensile Strength Retention for 7075 Aluminum. (Fig 3.7.4.1.1(c) from Mil-Hdbk-5H).

Similarly, use a curve for modulus retention and construct a similar function.



Modulus Retention for 7075 Aluminum. (Fig 3.7.4.1.4 Mil-Hdbk-5H)

For steel, assume a constant strength. To cover the possible temperature range, the reference strength was multiplied by 0.9. For Aluminum, start with a reference strength of 70 ksi and multiple by the retention factor as a function of temperature.

The strength of the combination is computed by assuming both components have the same strain. In this approach, the aluminum is not allowed to “fail” in the sense of exceeding the ultimate strength. In reality, at very high temperatures the aluminum could plastically deform and completely unload into the steel. This behavior is a bit complicated to model, so a more conservative strength model is used instead.

The composite electrical resistance and density are computed by simple weighted sums.

Power Considerations

The specific power for a large solar cell array (kW/kg) will determine the power unit mass. The value of 676 w/kg, from http://www.emcore.com/assets/photovoltaics/Paper_Navid_9-22-00.pdf. I’ve seen higher numbers (1003 kw/kg at http://www.orbital-power.com/home/thin-film-solar-cells/) for space solar power satellite studies. However, current space systems are much heavier. The specific power for the ISS solar arrays is only about 30 kg/kw (see http://www.shuttlepresskit.com/STS-97/payload81.htm)

```
specificPower = 676;
```

The propulsion scheme uses current flowing one way through the cable. The current loop is completed by capturing electrons at one end, and ejecting electrons at the other end. There is some efficiency loss associated with this part of the current loop. There are some research papers on this topic, but for now use a made-up efficiency factor of 0.85. This is applied to the power needed for propulsion.

```
returnCurrentEfficiency = 0.85;
```

The coefficient of drag applies to the cable. Assume a value of 1. It is possible for the coefficient to be greater than one if the molecules bounce off the cable.

```
coeffDrag = 1;
```

The solar power unit at the top of the system will be in the earth’s shadow for part of every orbit. We will assume that the drag is constant, but the momentum loss must be restored only during the daylight hours. Thus, the daytime power requirement is greater than would be needed if the drive operated continuously. We assume the system is massive enough relative to the drag loss that a cycle of turning off the propulsion during nighttime is acceptable.

An idea to be examined later: In order to keep the collector at a constant altitude during the nighttime momentum loss, consider a reel system that stores momentum by changing the free length of the cable.

Initial estimate of drag and power

Consider a cable 100 km long, and a collection altitude of 100 km.

```
cableLength = 100 × 10^3;
collectorAltitude = 100 × 10^3;
```

Estimate the system center-of-mass (assuming the mass at the top and bottom nodes are equal), and the result orbital angular velocity.

```
cm = rearth + collectorAltitude + cableLength / 2;
ω = Sqrt[μ / cm^3];
```

From these, we can compute the velocity at the capture device.

```
vcapture = (collectorAltitude + rearth) * ω Meter / Second
```

$$\frac{7755.47\ \text{Meter}}{\text{Second}}$$

If we assume that the device collects 1000 kg/day of gas, then the collection rate is

```
massRate = 1000. / (24 * 3600) Kilo Gram / Second
```

```
0.0115741 Gram Kilo
───────────────────
       Second
```

The collection rate times the velocity give the drag force for the collector.

```
collectorDrag = Convert[massRate * vcapture, Newton]
```

```
89.7624 Newton
```

Force times velocity gives the power needed to propel the capture device, not counting various system losses.

```
powerDrag = Convert[collectorDrag * vcapture, Watt]
```

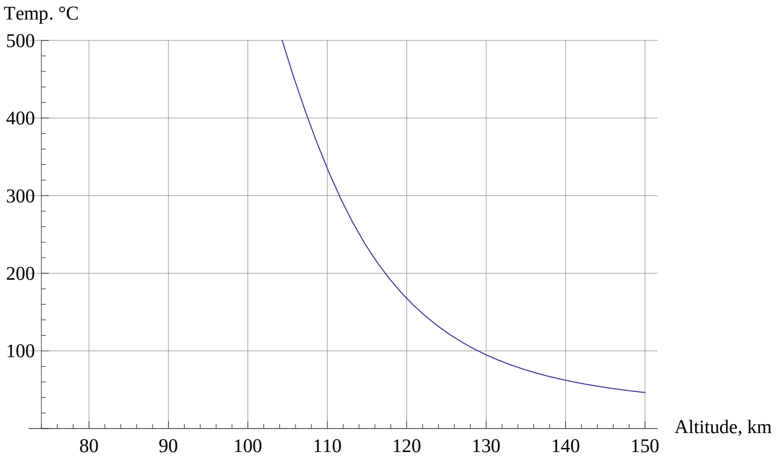
```
696 150. Watt
```

Operating Altitude

Assume that the altitude of the capture unit is determined by the maximum allowable temperature of the cable. The heat loads into the cable consists of aerodynamic heating, solar heating, and dissipated heat from the electrical current. The heat is carried away by radiation. We assume that because the cable is nearly vertical, there is no heat load from the earth.

The aerodynamic model uses H=1/2 ρ V^3 d, where ρ is the density of the atmosphere, and d is the cable diameter. The model assumes that all of the gas molecules deposit their kinetic energy onto the cable, which is probably very conservative. The solar heating model simply assumes that the cable is exposed to the solar energy load of 1387 W/m^2, so the heat load into the cable is 1387 d. This assumes that the radiation absorption coefficient is 1.0, but this is compatible with the emissivity assumption of 1.0 used in the radiation model, unless there is a coating that selects for the radiation wavelength.

The radiation model simply uses the Stefan–Boltzmann law, H= 2 π d σ T^4. The factor of 2π d accounts for the surface are of the cable. σ is the Stefan–Boltzmann constant, and T is the cable equilibrium temperature. This assumes that the cable is radiating to absolute zero in space. Not quite true, but should be an approximation.



Cable equilibrium temperature versus altitude.

Steel/Aluminum cable with 115 km capture altitude

The aluminum conductor has almost zero strength at 300 C. We can mitigate this loss of strength by adding some steel to the cable. If we target 200C for aerodynamic and solar heating alone, we are left with some room for electrical heating. Guess at an altitude of 115 km.

```
cableTemperature[115, 1, 0 ]
```

```
232.164
```

This will be the capture altitude.

```
captureAltitude = 115;
```

Given an altitude governed by the maximum operating temperature, we can now compute the capture area. First, find the density of the gas the capture altitude.

```
captureDensity = atmosphereDensityJa77["Total", 1000][captureAltitude]
  Kilo Gram / Meter^3
```

$$\frac{4.38514 \times 10^{-8}\ \text{Gram Kilo}}{\text{Meter}^3}$$

The required area is then

```
captureArea = massRate / captureDensity / vcapture
```

$34.0326\ \text{Meter}^2$

The equivalent circular diameter is

```
captureDia = 2 Sqrt[captureArea / Pi] // PowerExpand
```

```
6.58268 Meter
```

Unfortunately, I have not studied the mechanism needed to capture and liquefy 1000 kg/day of hypersonic gas, and therefore I do not have any way to come up with a mass estimate for the collector. For the purpose of moving forward, I’ve assigned a mass of 5000 kg to the collector because that “feels” like the minimum that would be needed to construct such a device.

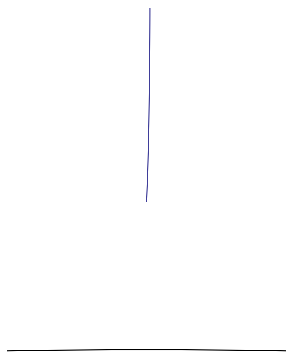
```
collectorMass = 5000;
```

Some design examples

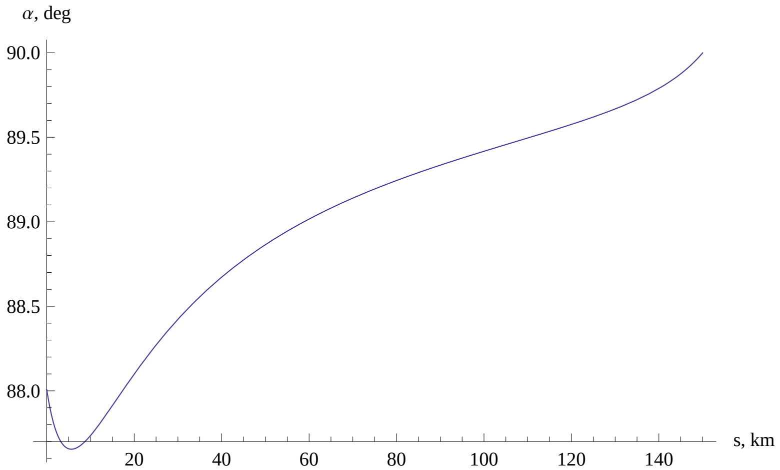
For the following, the cable diameter is adjusted by hand until the strength factor-of-safety (F.O.S) is at least 2. The increments for cable diameter trials was .05 cm. There is a table entry labeled “Top Mass Check”. This is a comparision of the calculated compatible top-node mass with the powerplant mass. The node mass must be greater than the powerplant mass. If not, the solution is increase the bottom node mass above the minumum assumed value for the collector mass. This forces the compatible top node mass to increase. The bottom-node mass was changed in 1000 kg increments, so again the results represent a very rough optimization.

Design results for 115 km collection altitude, 150 km cable length, and 60% aluminum cable.

Cable Length	150 Kilo Meter
Cable Dia	1.05 Centi Meter
Cable Drag	132.725 Newton
Collector Drag	89.1496 Newton
Power to Overcome Drag	1.70899×10^{6} Watt
Drive Voltage Drop	37 039.1 Volt
Resistance Voltage Drop	20 110. Volt
Current	46.1402 Amp
Total Power	4.97866×10^{6} Watt
Cable Mass	61 825.4 Gram Kilo
Powerplant Mass	7364.89 Gram Kilo
Bottom Node Mass	8000 Gram Kilo
Top Node Mass	7883.67 Gram Kilo
Total System Mass	77 709. Gram Kilo
Strength F.O.S	2.0113
Top Mass Check	True
Cable Max. Temp	238.453
Capture Area	34.2665 Meter^{2}



Cable path with segment of earth surface

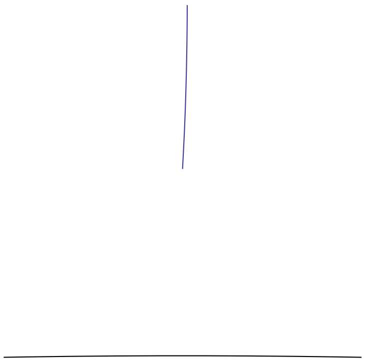


Cable angle relative to the local tangential direction.

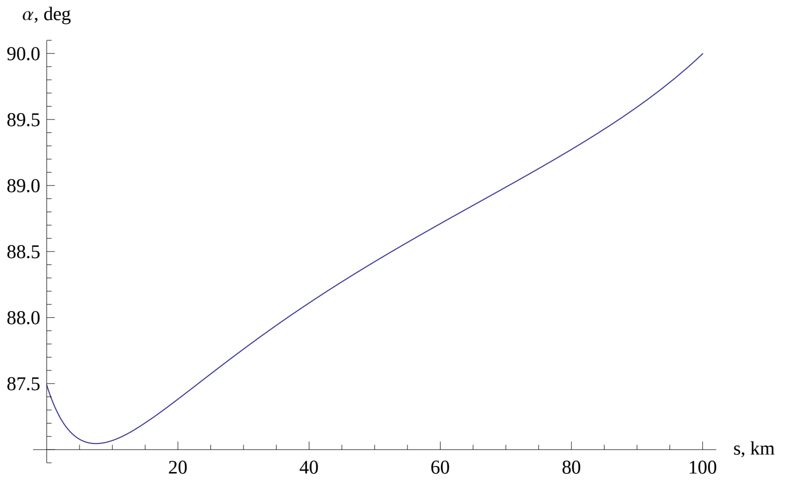
Design results for 115 km collection altitude, 100 km cable length, and 60% aluminum cable.

```
ans = designer[115, 1000, 1, 100 , .0070, 9500, .6]; ans[[1]]
```

```
Cable Length             | 100 Kilo Meter
Cable Dia                | 0.7 Centi Meter
Cable Drag               | 88.4696 Newton
Collector Drag           | 89.661 Newton
Power to Overcome Drag   | 1.37993 × 10^6 Watt
Drive Voltage Drop       | 25 116.5 Volt
Resistance Voltage Drop  | 35 918.7 Volt
Current                  | 54.941 Amp
Total Power              | 6.18534 × 10^6 Watt
Cable Mass               | 18 318.6 Gram Kilo
Powerplant Mass          | 9149.91 Gram Kilo
Bottom Node Mass         | 9500 Gram Kilo
Top Node Mass            | 9256.45 Gram Kilo
Total System Mass        | 37 075.1 Gram Kilo
Strength F.O.S           | 2.10494
Top Mass Check           | True
Cable Max. Temp          | 260.375
Capture Area             | 34.071 Meter^2
```



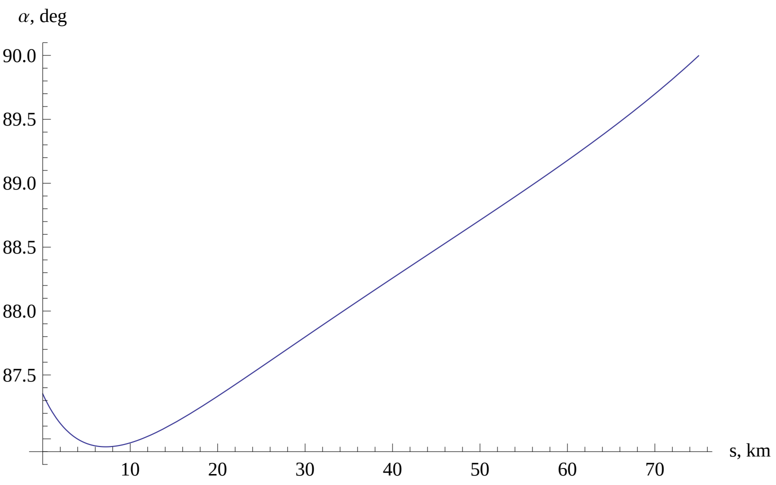
Cable path with segment of earth surface



Cable angle relative to the local tangential direction.

Design results for 115 km collection altitude, 75 km cable length, and 60% aluminum cable.

Cable Length	75 Kilo Meter
Cable Dia	0.65 Centi Meter
Cable Drag	81.3209 Newton
Collector Drag	89.9186 Newton
Power to Overcome Drag	1.33035×10^6 Watt
Drive Voltage Drop	18 999.2 Volt
Resistance Voltage Drop	39 818.6 Volt
Current	70.0215 Amp
Total Power	7.54796×10^6 Watt
Cable Mass	11 846.4 Gram Kilo
Powerplant Mass	11 165.6 Gram Kilo
Bottom Node Mass	12 000 Gram Kilo
Top Node Mass	11 566.1 Gram Kilo
Total System Mass	35 412.5 Gram Kilo
Strength F.O.S	2.12562
Top Mass Check	True
Cable Max. Temp	285.364
Capture Area	33.9735 Meter^2



Cable angle relative to the local tangential direction.

Design results for 115 km collection altitude, 50 km cable length, and 90% aluminum cable.

For this cable length, it was difficult to find a suitable design for the 60% aluminum combination. A 90% aluminum cable cable yielded a feasible design.

```
Cable Length              |50 Kilo Meter
Cable Dia                 |0.7 Centi Meter
Cable Drag                |84.7648 Newton
Collector Drag            |90.1774 Newton
Power to Overcome Drag    |1.36303 × 10^6 Watt
Drive Voltage Drop        |12 775.4 Volt
Resistance Voltage Drop   |11 422. Volt
Current                   |106.692 Amp
Total Power               |4.93707 × 10^6 Watt
Cable Mass                |6186.38 Gram Kilo
Powerplant Mass           |7303.36 Gram Kilo
Bottom Node Mass          |8000 Gram Kilo
Top Node Mass             |7349.09 Gram Kilo
Total System Mass         |21 535.5 Gram Kilo
Strength F.O.S            |3.36712
Top Mass Check            |True
Cable Max. Temp           |266.396
Capture Area              |33.876 Meter^2
```

All Aluminum cable with 125 km capture altitude

The aluminum conductor has almost zero strength at 300 C. We can mitigate this loss of strength by adding some steel to the cable. If we target 120C for aerodynamic and solar heating alone, we are left with some room for electrical heating. Guess at an altitude of 115 km.

```
cableTemperature[125, 1, 0 ]
```

```
123.891
```

This will be the capture altitude.

```
captureAltitude = 125;
```

Given an altitude governed by the maximum operating temperature, we can now compute the capture area. First, find the density of the gas the capture altitude.

```
captureDensity = atmosphereDensityJa77["Total", 1000][captureAltitude]
  Kilo Gram / Meter^3
```

$$\frac{1.28842\times 10^{-8}\ \text{Gram Kilo}}{\text{Meter}^{3}}$$

The required area is then

```
captureArea = massRate / captureDensity / vcapture
```

$115.385\ \text{Meter}^{2}$

The equivalent circular diameter is

```
captureDia = 2 Sqrt[captureArea / Pi] // PowerExpand
```

```
12.1208 Meter
```

Keep the collector mass the same.

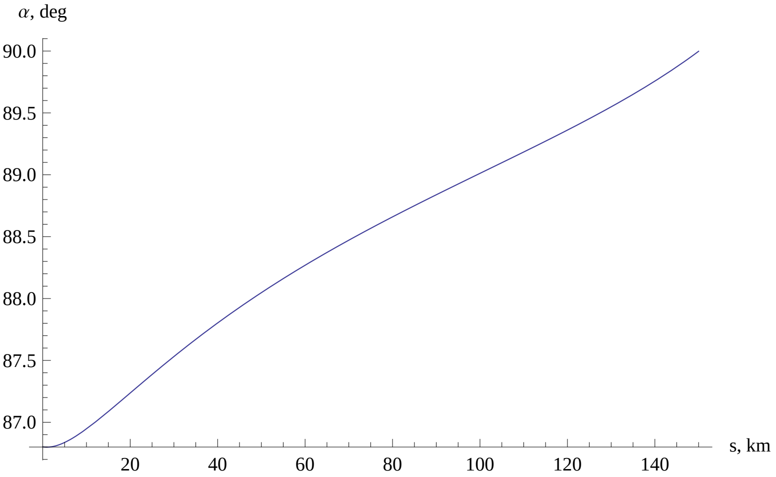
```
collectorMass = 5000;
```

Some solutions

Perform a similar series of design iterations for the higher collection altitude. The same hand adjustments are made as for the previous case of a mixed steel/aluminum cable.

Design results for 125 km collection altitude, 150 km cable length, and 100% aluminum cable.

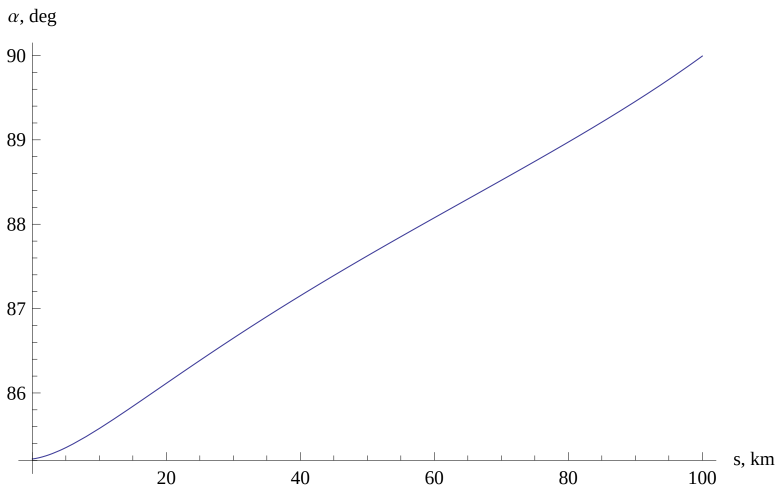
Cable Length	150 Kilo Meter
Cable Dia	0.5 Centi Meter
Cable Drag	26.8612 Newton
Collector Drag	89.0834 Newton
Power to Overcome Drag	892 403. Watt
Drive Voltage Drop	36 843. Volt
Resistance Voltage Drop	4811.05 Volt
Current	24.2217 Amp
Total Power	1.97084×10^{6} Watt
Cable Mass	7952.16 Gram Kilo
Powerplant Mass	2915.44 Gram Kilo
Bottom Node Mass	5000 Gram Kilo
Top Node Mass	4799.22 Gram Kilo
Total System Mass	17 751.4 Gram Kilo
Strength F.O.S	2.04352
Top Mass Check	True
Cable Max. Temp	126.565
Capture Area	$116.713\ \text{Meter}^{2}$



Cable angle relative to the local tangential direction.

Design results for 125 km collection altitude, 100 km cable length, and 100% aluminum cable.

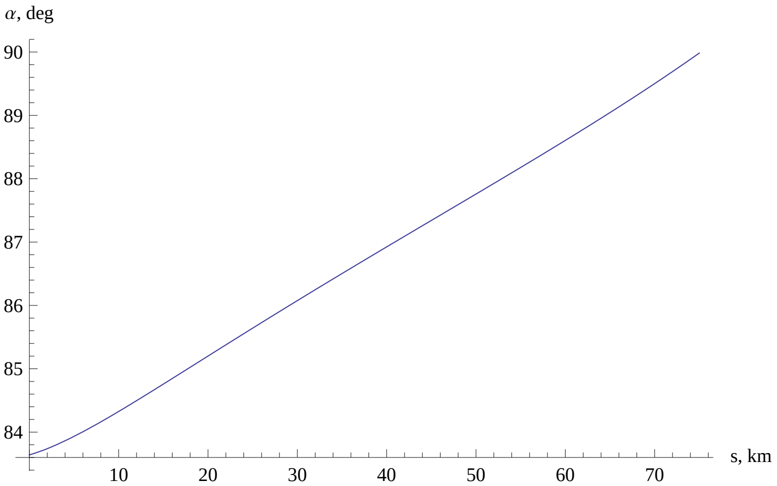
Cable Length	100 Kilo Meter
Cable Dia	0.45 Centi Meter
Cable Drag	23.942 Newton
Collector Drag	89.5937 Newton
Power to Overcome Drag	878 868. Watt
Drive Voltage Drop	24 982.9 Volt
Resistance Voltage Drop	5750.93 Volt
Current	35.1787 Amp
Total Power	2.11932×10^{6} Watt
Cable Mass	4294.16 Gram Kilo
Powerplant Mass	3135.09 Gram Kilo
Bottom Node Mass	5000 Gram Kilo
Top Node Mass	4709.61 Gram Kilo
Total System Mass	14 003.8 Gram Kilo
Strength F.O.S	2.55919
Top Mass Check	True
Cable Max. Temp	132.879
Capture Area	116.048 Meter^{2}



Cable angle relative to the local tangential direction.

Design results for 125 km collection altitude, 75 km cable length, and 100% aluminum cable.

Cable Length	75 Kilo Meter
Cable Dia	0.4 Centi Meter
Cable Drag	20.814 Newton
Collector Drag	89.8506 Newton
Power to Overcome Drag	859 100. Watt
Drive Voltage Drop	18 897.9 Volt
Resistance Voltage Drop	7054.31 Volt
Current	45.46 Amp
Total Power	2.30069×10^{6} Watt
Cable Mass	2544.69 Gram Kilo
Powerplant Mass	3403.39 Gram Kilo
Bottom Node Mass	5000 Gram Kilo
Top Node Mass	4506.28 Gram Kilo
Total System Mass	12 051. Gram Kilo
Strength F.O.S	2.47131
Top Mass Check	True
Cable Max. Temp	145.284
Capture Area	115.716 Meter^{2}



Cable angle relative to the local tangential direction.

Design results for 125 km collection altitude, 50 km cable length, and 100% aluminum cable.

Cable Length	50 Kilo Meter
Cable Dia	0.4 Centi Meter
Cable Drag	19.6222 Newton
Collector Drag	90.1088 Newton
Power to Overcome Drag	854 300. Watt
Drive Voltage Drop	12 707.1 Volt
Resistance Voltage Drop	6955.02 Volt
Current	67.2303 Amp
Total Power	2.56681×10^{6} Watt
Cable Mass	1696.46 Gram Kilo
Powerplant Mass	3797.05 Gram Kilo
Bottom Node Mass	5000 Gram Kilo
Top Node Mass	3944.36 Gram Kilo
Total System Mass	10 640.8 Gram Kilo
Strength F.O.S	2.97874
Top Mass Check	True
Cable Max. Temp	167.722
Capture Area	115.385 Meter^{2}

Comparisons and Observations

The table below summarizes the total mass results. There is a definite mass advantage to using a higher capture altitude. For the 150 km alum/steel mix it was hard to find a compatible design; the lower node mass had to be increased dramatically to support the mass of the power array at the upper node, resulting in a high system.

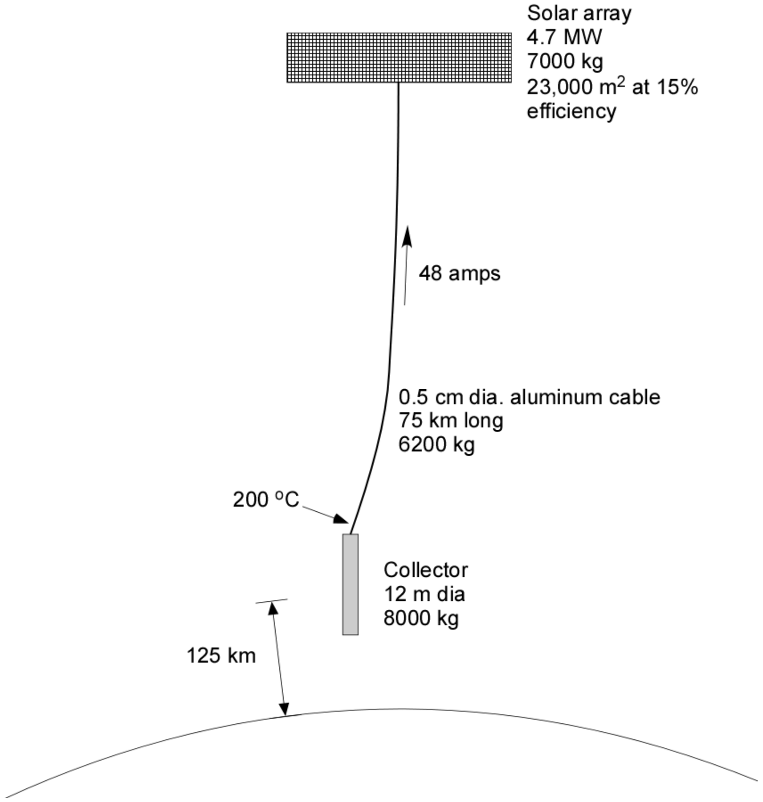
For the low-altitude system, the lower node mass had to be increased above the somewhat arbitrary minimum in order to balance the power array mass. If the collector mass is greater than our assumption, then there would be less penalty for using a lower altitude.

A reasonable system can be designed for a wide range of collection altitudes. This means that the selection will hinge more on the collection mechanism than on propulsion considerations. Lower altitudes may allow for continuum flow and use of ram compression. High altitude involves larger mean-free paths and atomic level collection. I have no insights on which may be preferred. This design study has gone into as much detail as is useful without further definition of the collector.

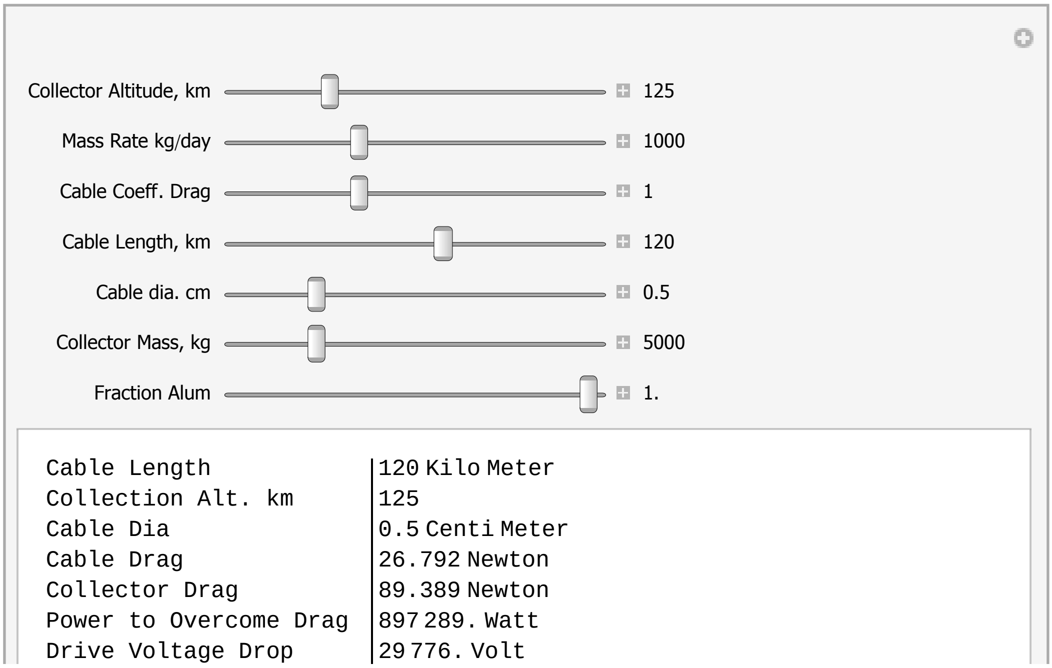
Cable Length	Alum / Steel	Alum
150	77 709.	17 751
100	37 075	14 004

75	35 412	12 051
50	21 535	10 641

The follow sketch summarizes one design case (75 km cable length, 125 km collection altitude) for quicker viewing.



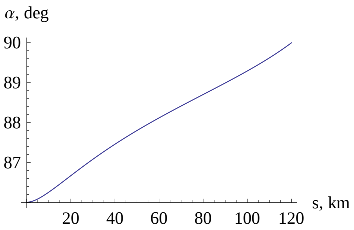
Interactive Design

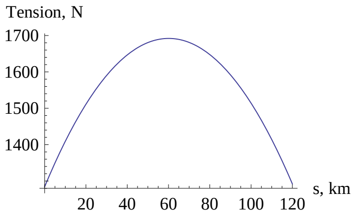


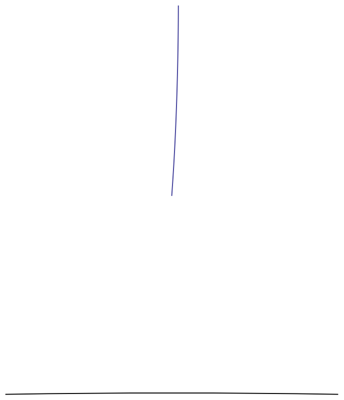
Collector Altitude, km 125
Mass Rate kg/day 1000
Cable Coeff. Drag 1
Cable Length, km 120
Cable dia. cm 0.5
Collector Mass, kg 5000
Fraction Alum 1.

```
Cable Length            120 Kilo Meter
Collection Alt. km      125
Cable Dia               0.5 Centi Meter
Cable Drag              26.792 Newton
Collector Drag          89.389 Newton
Power to Overcome Drag  897 289. Watt
Drive Voltage Drop      29 776. Volt
```

Resistance Voltage Drop	4788.4 Volt
Current	30.1346 Amp
Total Power	2.0447×10^6 Watt
Cable Mass	6361.73 Gram Kilo
Powerplant Mass	3024.71 Gram Kilo
Bottom Node Mass	5000 Gram Kilo
Top Node Mass	4783.55 Gram Kilo
Total System Mass	16 145.3 Gram Kilo
Strength F.O.S	2.6175
Top Mass Check	True
Cable Max. Temp	128.422
Capture Area	116.314 Meter^2







Other checks

Power to cover heat of vaporization of nitrogen and oxygen. N2=200 kJ/kg, O2 = 213 kJ/kg. Call the mix 206 kJ/kg so we don’t have to worry about the mixture ratio. For 1000 kg over 24 hrs, the power required is

```
206. * 1000 * 1000 / (24 * 3600)
```

```
2384.26
```

which is small compare to the megawatts needed to drive the system.

Ratio of oxygen to total for 115 km capture

```
0.296319
```

Ratio of oxygen to total for 125 km capture

```
0.363236
```