Initial Trajectory and Atmospheric Effects

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Introduction

A major consideration for an earth-based accelerator is atmospheric drag. Drag loses mean that the gun exit velocity must be higher so that there is sufficient remaining energy to reach the desired orbit. This study addresses the loss of velocity due to drag. Aerodynamic heating must also be considered, and this topic should be addressed in a later study. Of interest is the use of an accelerator to reach velocities such that only a minimum rocket burn will be required to circularize the orbit. For this purpose, a velocity at the outer edge of the atmosphere of 8-10 km sec is needed. An accelerator could also be usefully employed to provide a smaller fraction of the orbital energy, but this application is not examined in the work that follows. Similar trajectory calculations were performed as part of Ref. [1] and [2], for example.

The terms “ground-based accelerator” and “gun” will be used interchangeably. For the purposes of this study, the particular technology used for the acceleration is not important. We consider only the drag starting from the gun exit. It is assumed that the gun tube is evacuated during the acceleration phase.

There are a couple of strategies available to reduce drag loss. The first is to reduce the impact of drag on the projectile by increasing the ballistic coefficient. The effect of drag on a projectile depends on the projectile mass, which is included as a parameter in the ballistic coefficient. For this study, we will examine 1000 kg and 10,000 kg projectiles. The smaller projectile might be considered a lower limit on an autonomous vehicle that includes guidance, a rocket motor to circularize the orbit, thermal protection, and a practical payload. A second strategy is to build the gun such that the exit is at high altitude, and therefore above most of the atmosphere. To this end, we can consider the highlands of the near-equatorial region, mountains, and towers. Launch angle is yet another strategy. A high angle for the initial trajectory limits the distance transversed within the atmosphere. Finally, I’d like to examine an additional strategy that I have not seen published before; using a projectile with aerodynamic lift to change the trajectory path. The benefit is similar to using a steep launch angle, but the use of lift my allow for a horizontal or near horizontal gun, which should reduce the construction cost.

Model

The equations of motion for the projectile in Cartesian coordinates are given by

$$M \ddot{x} = -\frac{\mu M \cos(\alpha)}{r^2} - \frac{1}{2} A V^2 \rho(r) (C_L \sin(\psi) + C_D \cos(\psi))$$

$$M \ddot{y} = \frac{(-\mu) M \cos(\alpha)}{r^2} - \frac{1}{2} A V^2 \rho(r) (C_L \cos(\psi) - C_D \sin(\psi))$$

where μ is the gravitational constant times the earth’s mass, M is the projectile mass, A is the projectile cross-sectional area, V is the magnitude of the velocity vector, ρ(r) is the density of the atmosphere at radius r, C_L is the coefficient of lift, and C_D is the coefficient of drag. The angles α and ψ are defined in Figure 1. C_L and C_D are assumed to constant with respect to Mach number. The atmospheric density was computed using the standard model.

The equations of motion were integrated using the built-in *Mathematica* function NDSolve.

Note that initial velocity for the simulation is stated in terms of velocity relative to the gun exit. However, the total velocity entered as a starting condition includes the earth’s rotational speed, assuming a launch from the equator.

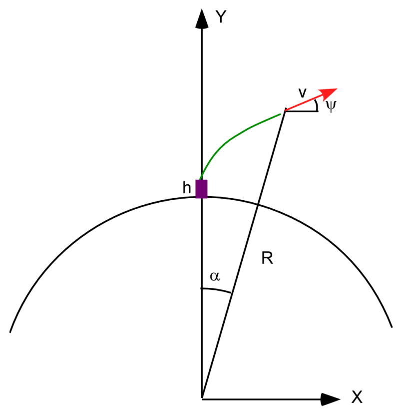

Figure 1. Coordinate system and angle definitions

Ballistic Coefficient

The traditional parameter that controls the free-flight of a projectile with drag is the ballistic parameter, usually defined as $M/(C_D$ A). I've found it useful to describe the projectile in terms of an average density, and a length-to-diameter ratio, $\bar{l}$. $\bar{l}$ is defined as the length of right cylinder with the same volume as the projectile, divided by the diameter. Using these parameters, it is possible to determine a reasonable area for a given mass. One can review existing rockets to determine a reasonable average density. After looking at data form some large liquid fuel rockets (examples, Ariane 5GP=723 kg$/m^3$, Saturn 5 first stage =650 kg$/m^3$), I decided that 750 kg$/m^3$ was a reasonable upper bound for density. Less clear is an appropriate length ratio. Looking at reentry vehicles (such as modern warheads) as a model, $\bar{l}$ should probably be 4 or less. On the other hand, Army hypersonic projectiles would be more in the range of $\bar{l}$ =10. The C_D and C_D values used in these studies roughly correspond to Newtonian flow results for a cone. These are probably not be applicable to a cone connected to a long cylinder. The studies below concentrate on $\bar{l}$=4, and $\bar{l}$=8 for as representative.

From the length and density parameters, mass is simply

$$M = A\,L\,\rho = \frac{1}{4}\,\pi\,d^3\,\rho\,\bar{l}$$

$$\text{or } d = \sqrt[3]{\frac{4\,M}{\pi\,\rho\,\bar{l}}}$$

$$\text{Then } A = \frac{\pi\,d^2}{4} = \left(\frac{\pi}{4}\right)^{1/3}\left(\frac{M}{\bar{l}\,\rho}\right)^{2/3}$$

And the ballistic coefficient is $C_B = \frac{1}{C_D}\left(\frac{4\,\bar{l}^2\,\rho^2\,M}{\pi}\right)^{1/3}$. Numerical values for this equation can be obtained from the manipulate widget in Figure 2.

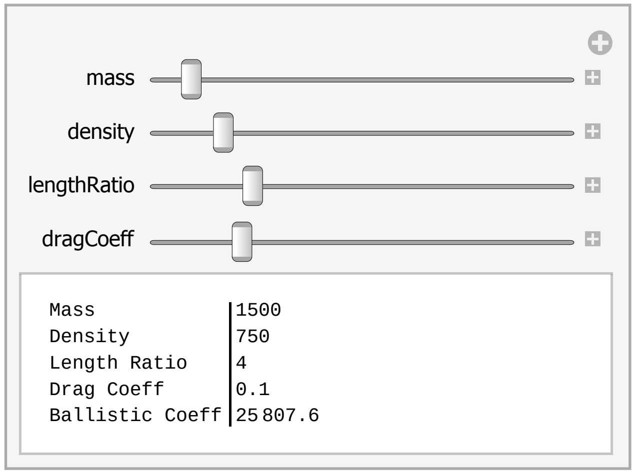

Figure 2. Manipulate for computing Ballistic Coefficient

In order to perform simulations of representative projectiles, a series of design cases has been defined, shown in Table 1. These cover two mass, and two length ratios. For Case 5, the density is also increased. The drag coefficient is held constant at 0.1. This appears to be an achievable value for a cone-shaped projectile at hypersonic speeds [3]. Reference 4 used a C_D is 0.09 for a "blunt-nose" case, and 0.044 for a "sharp-nose" case. The ballistic coefficient for Case 1 is on the same order as a modern nuclear warhead reentry vehicle [3].

Table 1. Series of projectile ballistic coefficients for design studies

	Mass, kg	Density, kg/m^3	Length/dia	Drag Coeff	Ballistic Coeff, kg/
Case 1	1000	750	4	0.1	22 545.
Case 2	1000	750	8	0.1	35 788.
Case 3	10 000	750	4	0.1	48 571.8
Case 4	10 000	750	8	0.1	77 102.9
Case 5	10 000	1000	8	0.1	93 403.5

Initial Deceleration

When the projectile leaves the gun atmospheric drag will immediately start to slow the projectile. The deceleration can be large and is one of the factors that may determine a minimum exit altitude. Thermal load is the other major factor, but that issue will not be discussed in this study. Figure 3 shows the deceleration as a function of altitude for a given set of design parameters. C_D is the coefficient of drag, λ is the length-to-diameter ratio of the projectile, ρ_P is the average density of the projectile, and $V_0$is the gun exit velocity. The plot range goes up to an altitude of 6 km, corresponding roughly to the height of mountains available. The deceleration of 200 g's at sea level is large, but acceptable compared to a gun acceleration of 500 g's or greater. Using the highest mountain halves the deceleration. Figure 4 is a widget for manipulating all the factors that affect the initial acceleration.

Note that these results to not include acceleration normal to the flight path provided by lift. If the lift over drag (L/D) ratio is greater than 1, then the normal acceleration will be greater than the longitudinal acceleration. The projectile equipment must be designed for the vector sum of the two components.

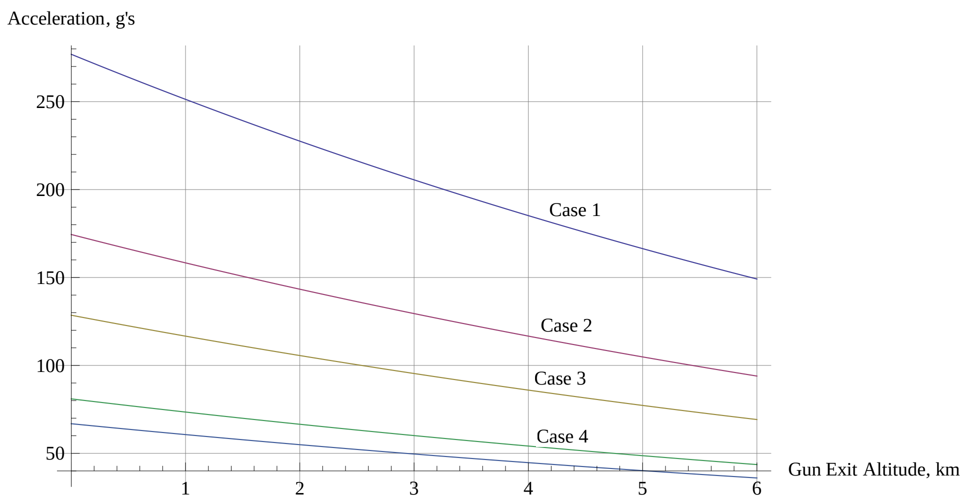

Figure 3. Initial deceleration at gun exit as a function of exit altitude. Cases are from Figure 2

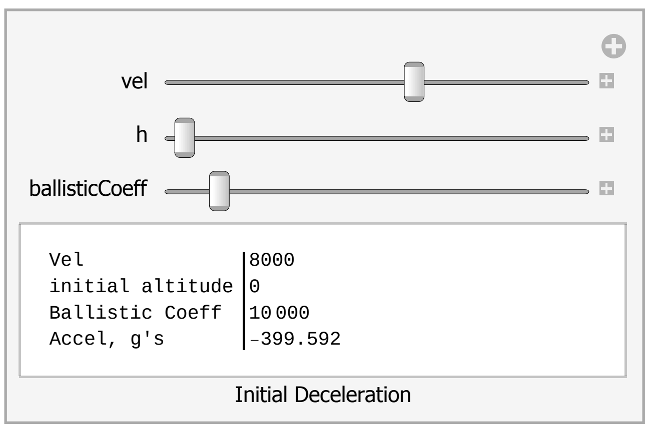

Figure 4. Manipulate widget for determining initial deceleration

Trajectory with Lift

Consider a projectile that can produce lift. The projectile can be as simple as a cone that establishes an angle-of-attack immediately upon gun exit. Maneuvering reentry vehicles provide a baseline input for this concept. From examples of hypersonic reentry vehicles and other simple shapes, it appears that a lift-to-drag ratio (L/D) up to 1.5 should be feasible [3]. Hypersonic lifting body designs reach 2.5 [5]. Figure 5 shows the initial flight path for a projectile with L/D=1.5 being launched from a horizontal gun. The plot shows that the lift can effectively change the flight path. Figure 6 shows the flight-path angle (with respect to the earth surface) as a function of time. The studies shown in later section will demonstrate that a minimum gun exit angle of 30° is needed to reduce aerodynamic losses to an acceptable level. Lift can be used to achieve a similar effect. Figure 7 shows the velocity magnitude as a function of time for this same trajectory and Figure 8 shows the acceleration on the projectile.

Using lift would also add operational flexibility because the lift could be used to change orbital plane to some degree. The two-dimensional simulation being used here does not allow us to examine quantify this possibility.

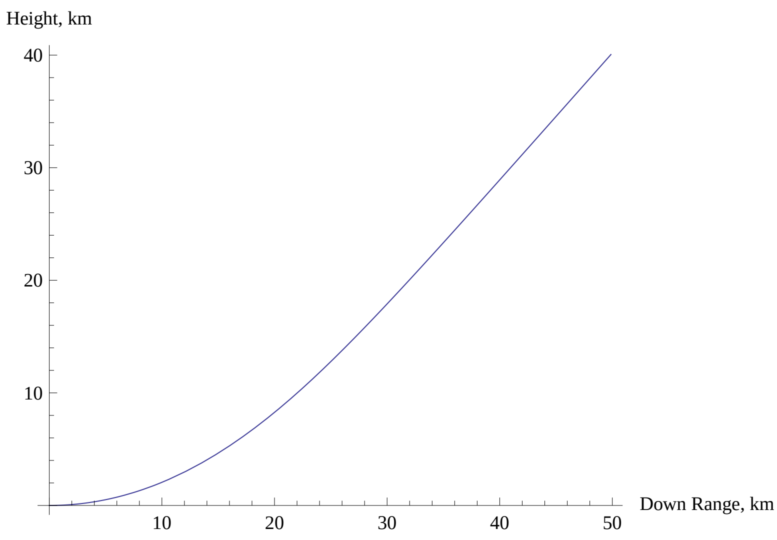

Figure 5. Trajectory for Case 1 projectile with L/D=1.5. Sea-level, horizontal launch at 10 km/sec

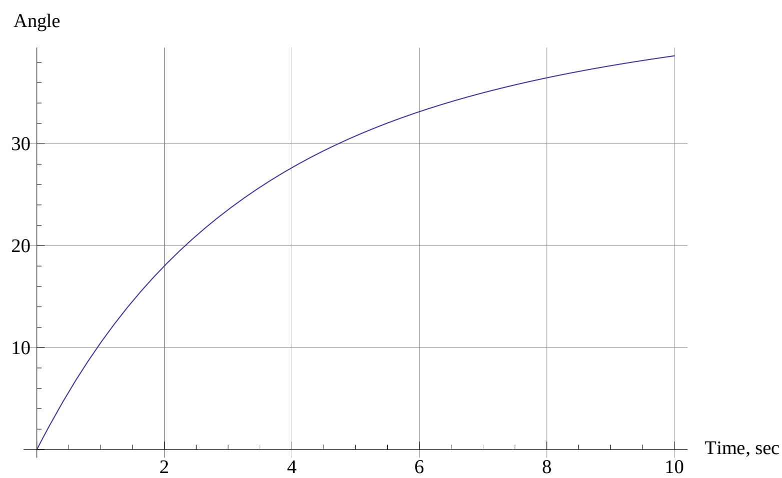

Figure 6. Flight path angle relative to earth surface. Case 1 projectile with L/D=1.5. Sea-level, horizontal launch at 10 km/sec

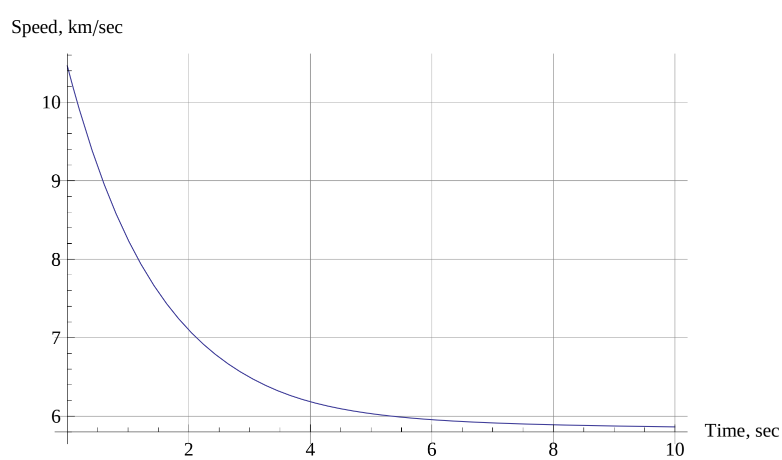

Figure 7. Projectile resultant velocity as function of time. Case 1 projectile with L/D=1.5. Sea-level, horizontal launch at 10 km/sec

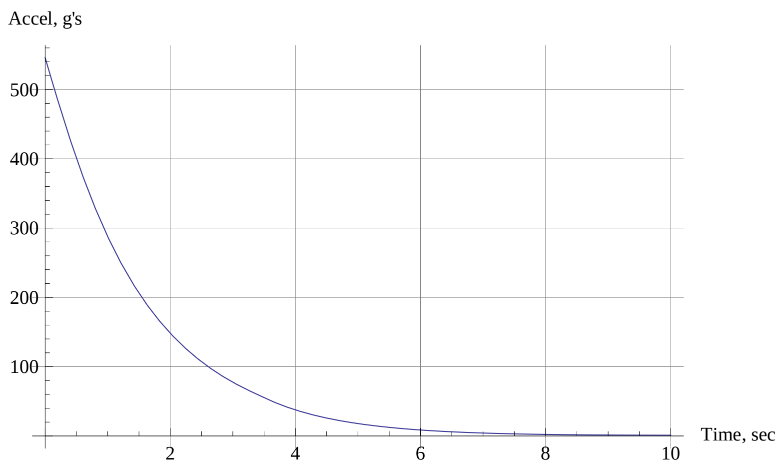

Figure 8. Projectile resultant acceleration as function of time. Case 1 projectile with L/D=1.5. Sea-level, horizontal launch at 10 km/s

Velocity at Edge of Atmosphere

For insertion into orbit, the important parameter is the velocity at the edge of the sensible atmosphere. For this study, the velocity will be computed for an altitude of 50 km. In determining the rocket impulse needed to achieve a desired orbit, the gun launch velocity vector is important. However, for the purposes of this study, we will look at only the velocity magnitude as a metric for determining aerodynamic losses. The goal is more to guide the gun design and site selection rather than devise specific trajectories to orbit. Figure 9 gives a manipulate widget that will compute the velocity magnitude at 50 km. The widget uses a numerical solution to first determine the time to reach 50 km, and then evaluates the trajectory solution at that time.

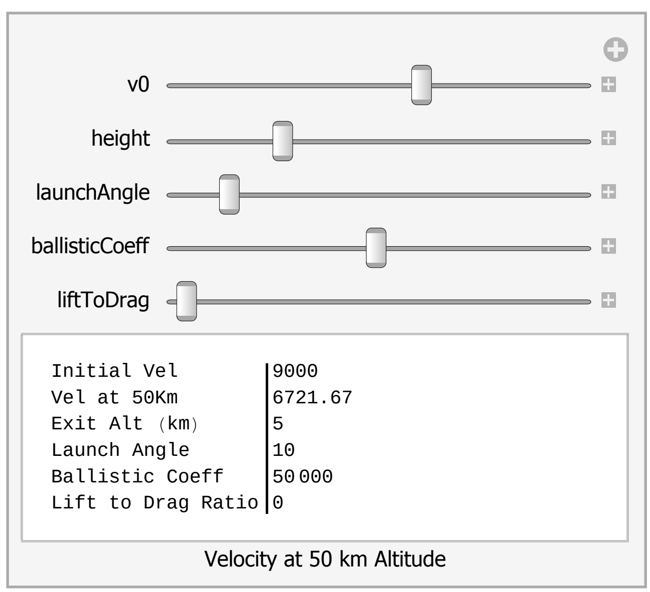

Figure 9. Manipulate for computing magnitude of velocity at 50 km altitude.

No-lift case

The trajectory problem can be inverted to a more direct design question: Given a desired velocity upon exiting the atmosphere, what is the required gun exit velocity? Figures 10 and 11 show the required velocity needed to achieve 8 km/sec at altitude, as a function of gun exit angle. Figure 10 is for a gun exit altitude at sea-level, and Figure 11 is for a gun exit altitude of 4 km. The plots show a rapid increase in the gun initial velocity for exit angles less than about 30°. Figure 12 and 13 repeat the same plots, but this time for a desired velocity at altitude of 10 km/sec. One way to use these charts is to decide on a maximum practical gun velocity, as driven by factors such as the gun technology limits, and aerothermo-heating. For example, a good design may limit aerodynamic losses to 20% of the initial velocity. This implies a target of 10 km/sec initial speed for an 8 km/sec speed at altitude, or a 12 km/sec speed to achieve 10 km/sec at altitude.

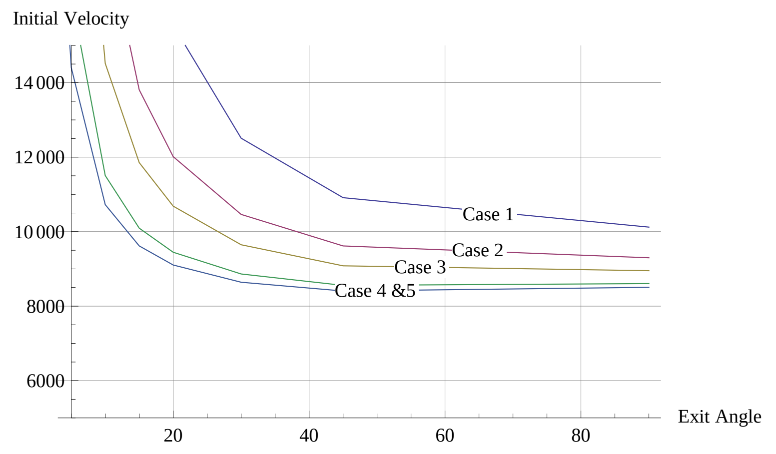

Figure 10. Initial velocity required so that projectile velocity is 8 km/sec at 50 km altitude. Gun exit at sea-level.

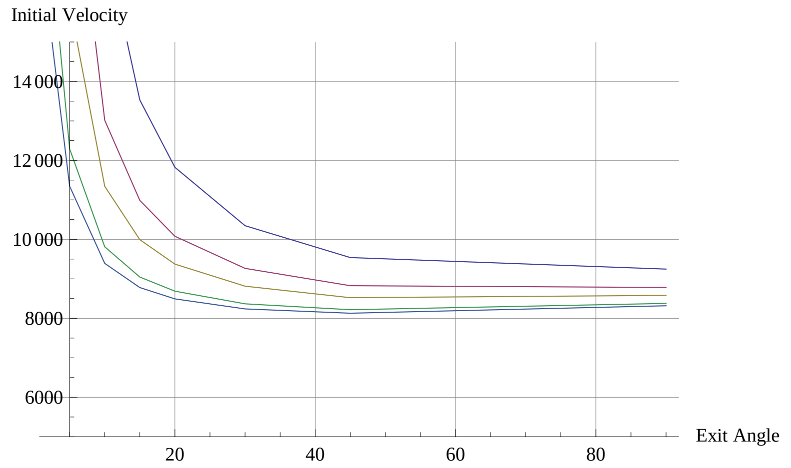

Figure 11. Initial velocity required so that projectile velocity is 8 km/sec at 50 km altitude. Gun exit at 4 km altitude.

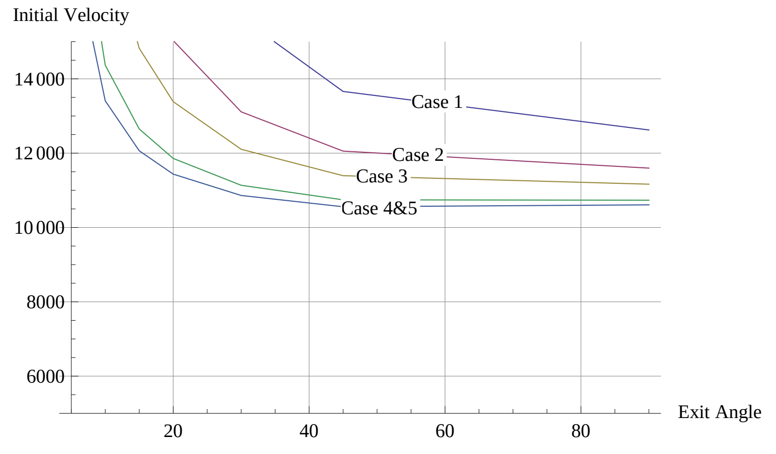

Figure 12. Initial velocity required so that projectile velocity is 10 km/sec at 50 km altitude. Gun exit at sea-level.

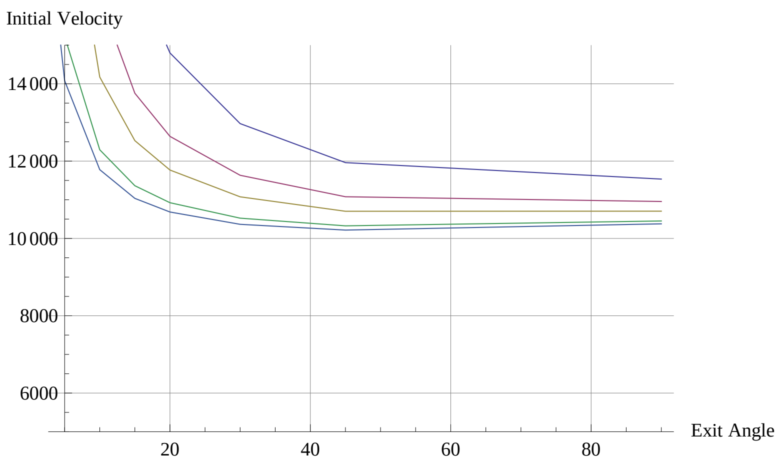

Figure 13. Initial velocity required so that projectile velocity is 10 km/sec at 50 km altitude. Gun exit at 4 km altitude.

Lift combined with a horizontal gun

Figures 14 and 15 examine the required gun velocity for a projectile with lift, this time as a function of gun exit altitude. Figure 14 is for the Case 1 ballistic coefficient, and Figure 15 is for Case 3. The longer projectiles (Case 2, 4 and 5) may not be as amenable to flying at a non-zero angle-of-attack. For these parameters, our stated (somewhat arbitrary) goal of limiting the gun exit velocity to 10 km/sec cannot be meet except for the Case 3 projectile launched at a very high altitude. An possible design scenario would be to have a horizontal gun at sea-level. For example, this would allow one to construct the gun in shallow water off the coast of French Guiana and avoiding any problems with flying over populated land. For the Case 3 projectile L/D=1.5, this would imply an exit velocity of apparently 11.5 km/sec.

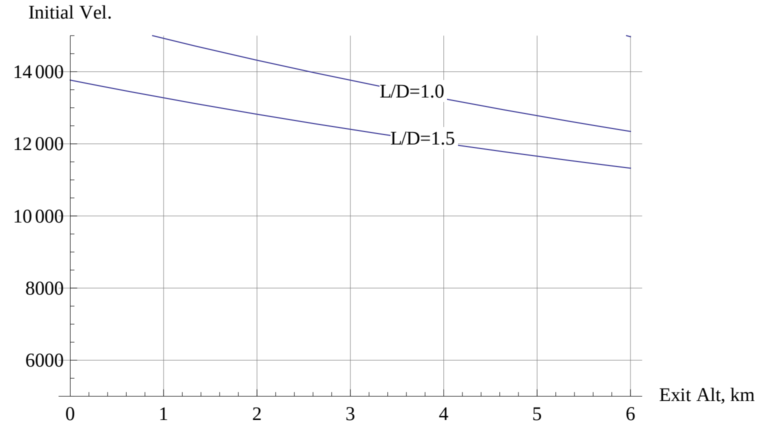

Figure 14. Initial velocity required so that projectile velocity is 8 km/sec at 50 km altitude for projectile with lift. Case 1 projectile.

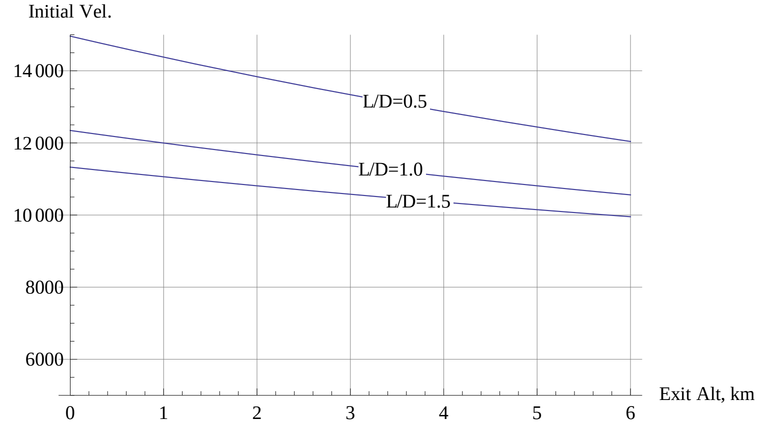

Figure 15. Initial velocity required so that projectile velocity is 8 km/sec at 50 km altitude for projectile with lift. Case 3 projectile.

Combined gun angle and lift

It will be challenging to find a nature site that allows for a long (10 km) accelerator combined with a steep angle. Even using mountains, extensive tunneling will be required. A shallow angle opens up more possibilities. One scheme is to curve the portion of the gun tube or track to change the launch angle. A quick design study showed that it may be practical to start with a horizontal track and curve to a 10° launch angle at the exit. The low launch angle can then be combined with lift to get a favorable trajectory. Tables 2-5 show initial launch velocities for various gun altitudes and L/D ratios. All of the tables assume a gun launch angle of 10°. Combining lift and a shallow gun angle allows for a significantly wider range of possibilities for designs with moderate aerodynamic losses.

Table 2. Initial velocity required so that projectile velocity is 8 km/sec at 50 km altitude. Case 1 projectile. Gun exit angle of 10°

Alt,km	L/D=0.0	L/D=0.5	L/D=1.0	L/D=1.5
0	29 265.8	15 527.8	13 405.9	12 426.
1	25 298.4	14 692.1	12 855.9	11 988.6
2	22 151.1	13 936.6	12 352.1	11 586.8
4	17 584.3	12 633.5	11 466.1	10 876.2
6	14 543.3	11 563.7	10 718.9	10 271.6

Table 3. Initial velocity required so that projectile velocity is 8 km/sec at 50 km altitude. Case 3 projectile. Gun exit angle of 10°

Alt,km	L/D=0.0	L/D=0.5	L/D=1.0	L/D=1.5
0	14 515.4	11 563.5	10 721.8	10 275.7
1	13 528.9	11 163.3	10 436.4	10 043.
2	12 686.5	10 795.9	10 171.2	9825.78
4	11 344.	10 149.8	9695.98	9433.73
6	10 347.7	9608.25	9287.17	9092.77

Table 4. Initial velocity required so that projectile velocity is 10 km/sec at 50 km altitude. Case 1 projectile. Gun exit angle of 10°

Alt,km	L/D=0.5	L/D=1.0	L/D=1.5
0	20 610.8	15 579.8	13 767.5
1	19 434.3	14 922.4	13 274.4
2	18 366.6	14 318.6	12 820.5
4	16 512.1	13 251.7	12 014.8
6	14 970.4	12 344.2	11 324.8

Table 5. Initial velocity required so that projectile velocity is 10 km/sec at 50 km altitude. Case 3 projectile.

Alt,km	L/D=0.5	L/D=1.0	L/D=1.5
0	14 960.8	12 345.5	11 328.5
1	14 377.1	11 996.	11 061.3
2	13 836.6	11 669.3	10 810.8
4	12 871.1	11 077.8	10 355.2
6	12 040.	10 559.8	9953.51

References

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