Full trajectory from gun exit to circular orbit

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Introduction

Two studies related to the gun projectile trajectory were previously posted. The first, “gun launch to circular.nb” used classical orbit mechanics equations to looks at various ways to use an earth based gun and achieve a circular orbit. The strategies investigated where a single-burn method, a two-burn method, and a method that included aerobraking. The study concluded that for most of the orbits of interest, a single-burn strategy required the lowest ΔV from the projectile rocket in order to achieve orbit. In this strategy, the gun exit velocity is adjusted so that the apogee of the projectile trajectory is at the desired orbit altitude. At the apogee, a rocket onboard the projectile burns in order to change the orbit to circular. In order for the gun to be beneficial, the ΔV of this rocket burn should be substantually less than the total ΔV required for a conventional rocket launch.

A second study, “trajectory model.nb” used a numerical integration of the flight equations to examine the velocity losses due to the atmospheric friction. This study concentrated on the upper limit of desirable gun exit velocities (8-10 km/sec). This study also examined the use of projectile lift to modify the trajectory to pass through the atmosphere quicker. Lift could be used in place of a fixed gun angle to minimize atmospheric losses.

At the time, the aurthor did not do a good job of merging these two studies. The gun exit velocity needed for a single-burn insertion to low-earth-orbit is substantially less than the upper bound used for the atmospheric loss study. Also, breaking the trajectory into two parts did not clearly indicate desirable values for launch angle, altitude, and projectile lift-to-drag ratio. The current study merges the two flight phases so that the gun parameters and rocket burn needed to reach a desired orbit can be calculated directly.

Studies

Cases for ballistic coefficient

The following cases for projectile parameters were copied directly from the study “trajectory model.nb”.

For the studies below, we will compute a range of ballistic coefficients for some practical designs. They range from (Case 1) a light (1000 kg) projectile with a modest length-to-diameter ratio (4), (Case 2) a light projectile with a longer length, (Case 3) a heavy projectile (10,000kg) with a length ratio of 4, (Case 4) a heavy projectile with a long length, and (Case 5) a long heavy projectile with a higher density.

Table 1. Some representative design cases for projectiles and their ballistic coefficients.

	Mass, kg	Density, kg/m^3	Length/dia	Drag Coeff	Ballistic Coeff, kg/
Case 1	1000	750	4	0.1	22 545.
Case 2	1000	750	8	0.1	35 788.
Case 3	10 000	750	4	0.1	48 571.8
Case 4	10 000	750	8	0.1	77 102.9
Case 5	10 000	1000	8	0.1	93 403.5

ballisticCoeffUndo takes a ballistic coefficient value as input, and returns a list {area,mass,dragCoeff} that will give the same coefficient. The list is not unique. The trajectory calculations depend only on the combined ballistic coefficient, so this a way to allow for single parameter input.

Low-Earth Orbit (250 km)

The mostly likely use for a gun assisted launch is for inserting mass to a low circular orbit. Two hundred and fifty kilometers has been selected as a representative orbit. Based on previous studies, we will consider only a single burn insertion to a circular orbit. This maneuver requires a rocket burn from the projectile at the apogee of the gun trajectory.

Perhaps the easiest system to build would be a horizontal gun at sea level. One possibility is to build the device in shallow waters off the coast. Figure 1 shows the required gun exit velocity as a function of the projectile lift-to-drag ratio. Two ballistic coefficient cases are examined; Case 1 is equivalent to 22500 kg/m^2, and Case 3 is equivalent to 49000 kg/m^2. The curves start at the minimum L/D that provides a valid solution. In other words, for Case 3 (red line), a minimum L/D of 1.2 is required. This might be hard to achieve with the shapes that are compatible with a gun launch and that can survive the extreme thermal environment. Also note that we have assumed a fixed coefficient of drag. The shapes that yield a high L/D may also increase the drag. Figure 2 shows that rocket ΔV required to circularize the orbit relative to the ΔV needed for a pure rocket launch (assuming zero drag and gravity losses). If the relatively high L/D values can be used, then it is feasible to launch from sea level with a horizontal gun, although the thermal loads have yet to be calculated.

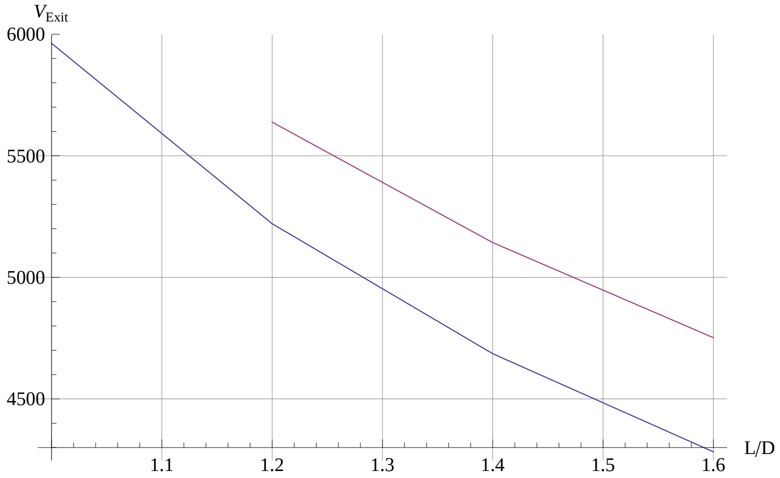

Figure 1. Gun exit velocity as function of lift-to-drag ratio. Exit angle of 0° at 0 m above sea level. Blue is for design case 1, and red is design case 3. Final orbit altitude of 250 km.

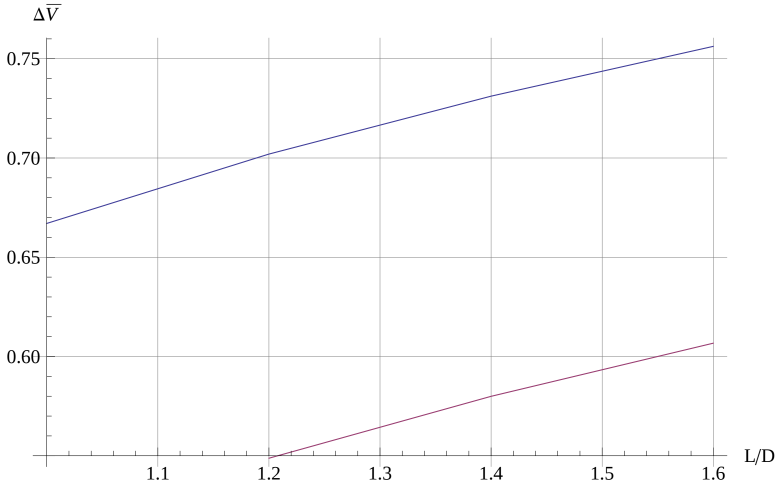

Figure 2. Gun assisted rocket ΔV relative to pure rocket ΔV as function of lift-to-drag ratio. Exit angle of 0° at 0 m above sea level. Blue is for design case 1, and red is design case 3. Final orbit altitude of 250 km.

Figures 3 and 4 repeat these calculates for a horizontal gun with the gun exit at 4000 m altitude. For the trajectory calculations, there is little advantage to building the device at high altitude. Again, thermal considerations may give a different conclusion. At high altitude the L/D requirements increase. This makes sense because the lower density air provides in a smaller change in trajectory when using lift to alter the trajectory.

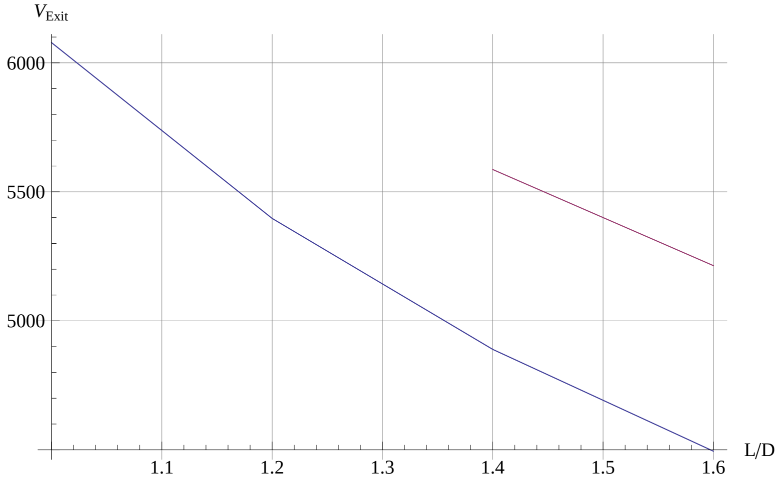

Figure 3. Gun exit velocity as function of lift-to-drag ratio. Exit angle of 0° at 0 m above sea level. Blue is for design case 1, and red is design case 3. Final orbit altitude of 250 km.

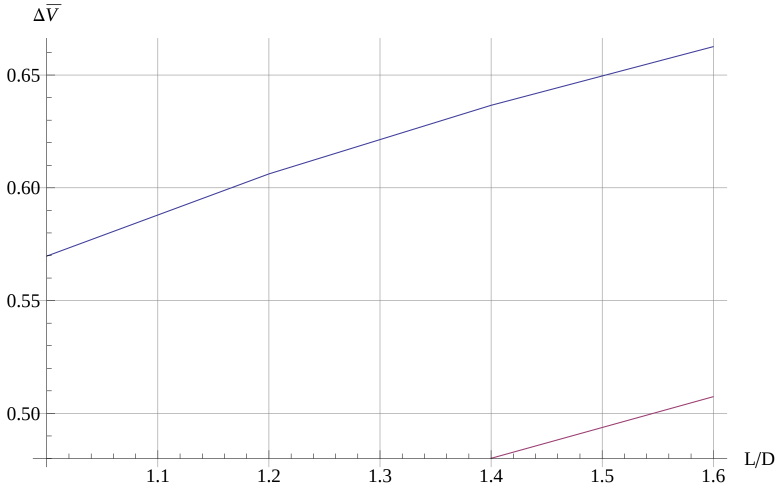

Figure 4. Gun assisted rocket ΔV relative to pure rocket ΔV as function of lift-to-drag ratio. Exit angle of 0° at 0 m above sea level. Blue is for design case 1, and red is design case 3. Final orbit altitude of 250 km.

Figure 5 and 6 consider the case of a gun with a 4000 m exit altitude, and an exit angle of 10°. In a previous study we examined the geometry and forces involved with using curved gun path change the exit angle and determined that modest angle changes are feasible. The biggest change that results from adding a gun exit angle is that there are now solutions for the Case 3 design down to a L/D of 1.0.

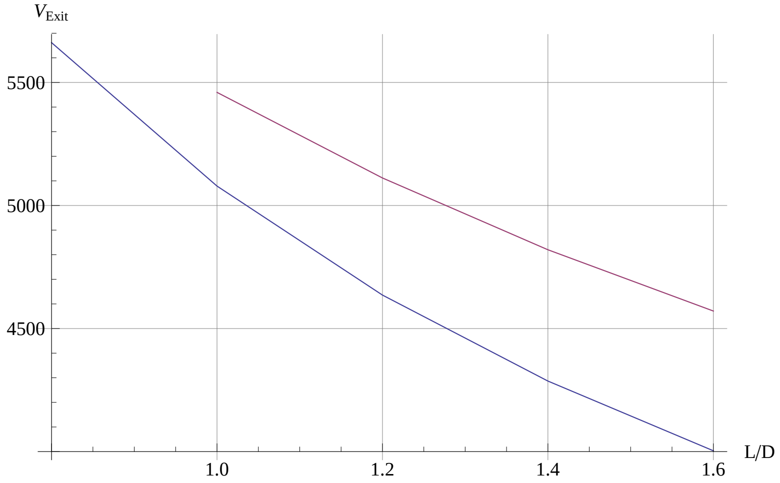

Figure 5. Gun exit velocity as function of lift-to-drag ratio. Exit angle of 10° at 4000 m above sea level. Blue is for design case 1, and red is design case 3. Final orbit altitude of 250 km.

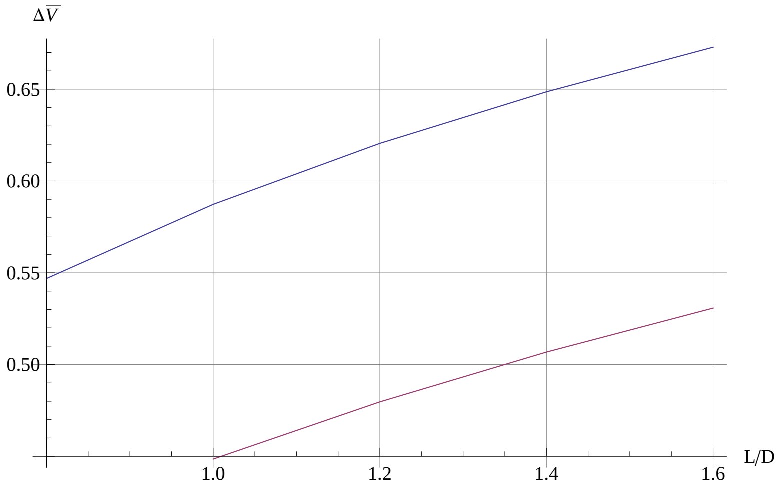

Figure 6. Gun assisted rocket ΔV relative to pure rocket ΔV as function of lift-to-drag ratio. Exit angle of 10° at 4000 m above sea level. Blue is for design case 1, and red is design case 3. Final orbit altitude of 250 km.

Finally, we consider higher gun angles and no lift. Removing lift would simplify the projectile design and control issues. The gun would be built up the side of a steep mountain. Figures 7 and 8 show the results for a 2000 m exit altitude. Building a gun with a 30° path eliminates the need for lift. Note that increasing the gun angle also increases the rocket ΔV needed to circularize the orbit.

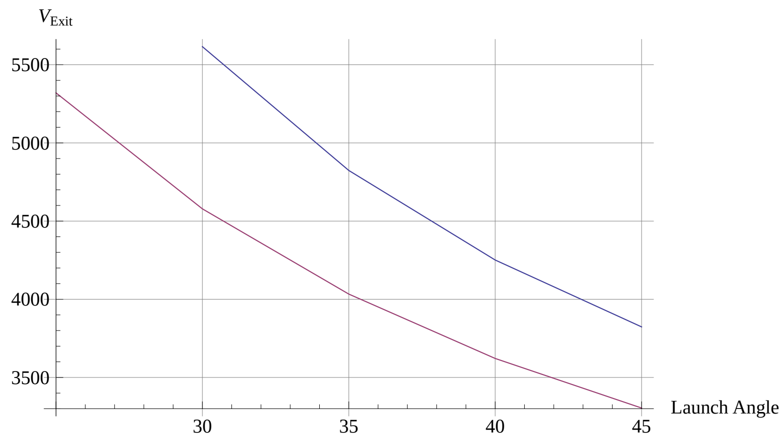

Figure 7. Gun exit velocity as function of launch angle. Gun exit at 2000 m above sea level. Blue is for design case 1, and red is design case 3. Final orbit altitude of 250 km.

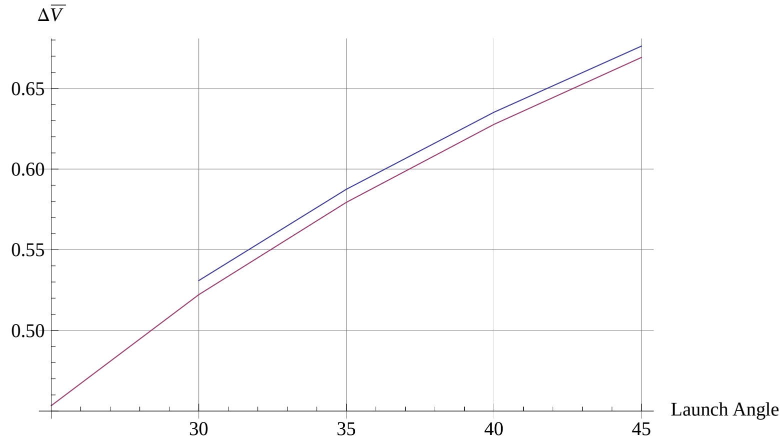

Figure 8. Gun assisted rocket ΔV relative to pure rocket ΔV as function of launch angle. Gun exit at 2000 m above sea level. Blue is for design case 1, and red is design case 3. Final orbit altitude of 250 km.

Higher Orbits

Gun assist also works for high altitude orbits. Figure 9 and 10 examine the case of a horizontal gun with an exit altitude of 2000 m and the Case 3 ballistic coefficient. An L/D of 1.6 is assumed. The exit velocities get quite large, but the relative ΔV grows small with increasing altitude. The upper bound for orbit altitude will be determined by the gun velocity limitations and the thermal limits for the projectile.

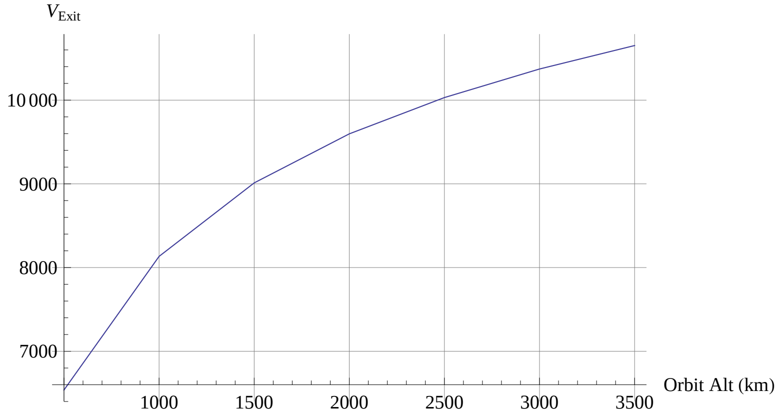

Figure 9. Gun exit velocity as function of final orbit altitude. Exit angle of 0° at 2000 m above sea level. Case 3 design with L/D=1.6

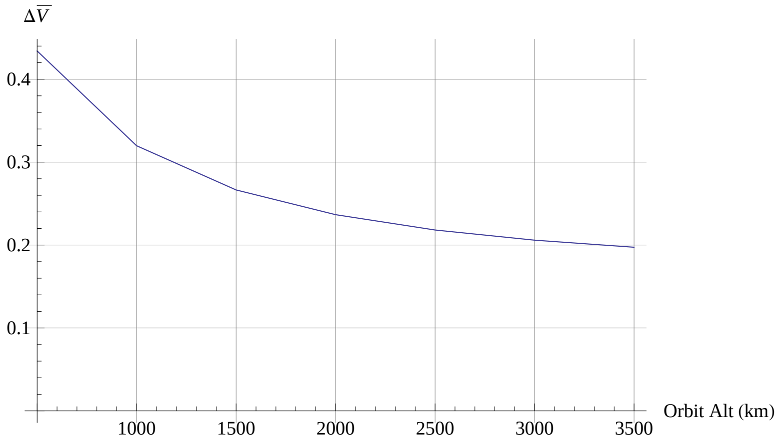

Figure 10. Gun assisted rocket ΔV relative to pure rocket ΔV as function of final orbit altitude. Exit angle of 10° at 2000 m. above sea level. Case 3 design with L/D=1.6

Finally, we'll consider a vertical launch. A vertical launch eliminates the need for lift and reduces the thermal load by exiting the atmosphere as quickly as possible. Figures 11 and 12 show the requirements for Case 2 and 4 designs. The long, thin cases would be more appropriate for a zero-lift condition. The gun exit velocities in Figure 11 are more modest than shown in Figure 9, but the rocket ΔV is not promising until orbits with at least a 4000 km altitude are needed.

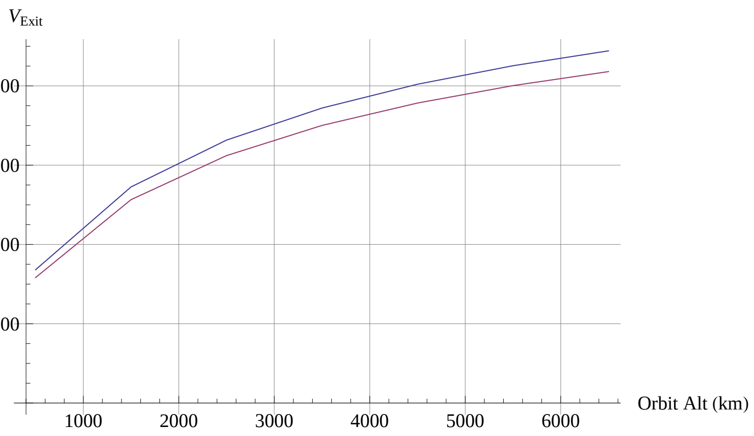

Figure 11. Gun exit velocity as function of final orbit altitude. Exit angle of 90 at 2000 m above sea level. Blue is for design case 2, and red is design case 4.

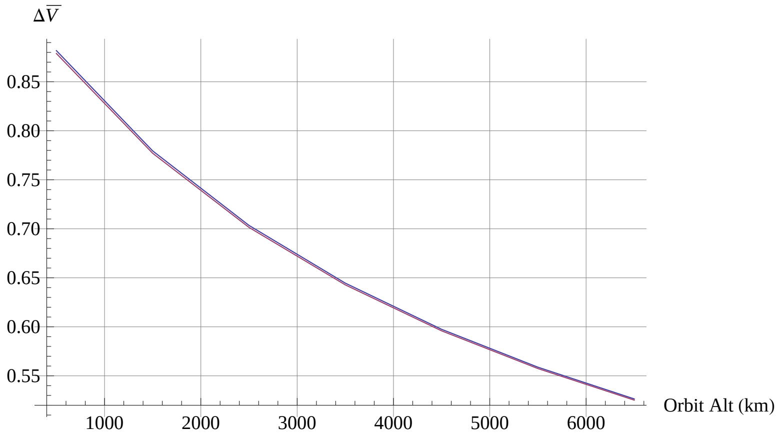

Figure 10. Gun assisted rocket ΔV relative to pure rocket ΔV as function of final orbit altitude. Exit angle of 90° at 2000 m. above sea level. Blue is for design case 2, and red is design case 4.